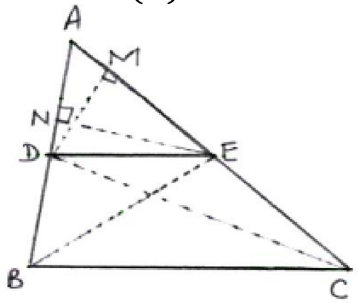
Triangles

1. The greater of two supplementary angles exceeds the smaller by 18°. Find measures of these two angles. (2024)

Answer. Let the measure of two angles be x° and y° (x > y)
Given x + y = 180 and x – y = 18
solving equations to get y = 81 and x = 99

2. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that other two sides are divided in the same ratio. (2024)

Answer. (A)



Given: In Δ ABC, DE || BC

To Prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Join BE, DC

Draw DM ⊥ AC and EN ⊥ AB

Proof: $\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2}\times AD\times EN}{\frac{1}{2}\times DB\times EN}$

$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{AD}{DB}$ ....(i)

and $\frac{ar(\Delta ADE)}{ar(\Delta CDE)} = \frac{\frac{1}{2}\times AE\times DM}{\frac{1}{2}\times EC\times DM}$

$\frac{ar(\Delta ADE)}{ar(\Delta CDE)} = \frac{AE}{EC}$ ....(ii)

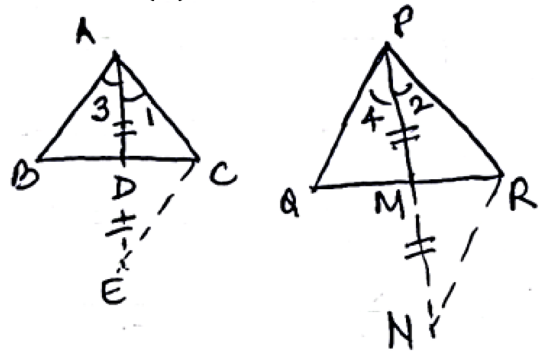
Δ BDE and Δ CDE are on the same base DE and between the same parallels DE and BC.

∴ ar (Δ BDE) = ar (Δ CDE) ....(iii)

From (i), (ii) and (iii)
$\frac{AD}{DB}=\frac{AE}{EC}$

3. Sides AB and BC and median AD of a AABC are respectively proportional to sides PQ and PR and median PM of APQR. Show that (2024)
AABC - APQR.

Answer. (B)Produce AD to E and PM to N such that AD = DE, PM = MN.



Δ ADB $\cong \Delta$ EDC $\Rightarrow$ AB = CE, similarly PQ = RN.

Given $\frac{AB}{PQ}=\frac{AC}{PR}=\frac{AD}{PM}$

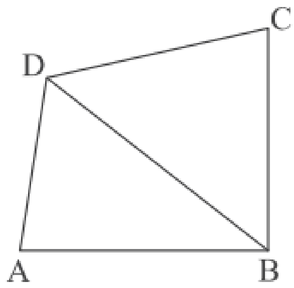
$$\Rightarrow \frac{CE}{RN}=\frac{AC}{PR}=\frac{\frac{AE}{2}}{\frac{PN}{2}} \Rightarrow \Delta AEC \sim \Delta PNR$$

$\Rightarrow \angle 1 = \angle 2$, similarly $\angle 3 = \angle 4$
therefore $\angle 1 + \angle 3 = \angle 2 + \angle 4$ or $\angle BAC = \angle QPR$

Also $\frac{AB}{PQ}=\frac{AC}{PR}$ (given)

Therefore $\Delta ABC \sim \Delta PQR$

4. In the given figure, ABCD is a quadrilateral.
Diagonal BD bisects ZB and D both. (2024)
Prove that:
(i) AABD ~ ACBD
(ii) AB = BC



Answer. (i) In ΔABD & ΔCBD

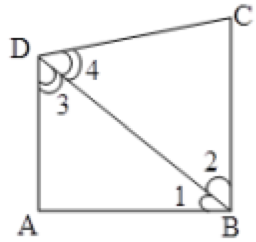
$\angle 3 = \angle 4$

$\angle 1 = \angle 2$

$\therefore \Delta ABD \sim \Delta CBD$

(ii) $\Delta ABD \cong \Delta CBD$

$\therefore AB = BC$

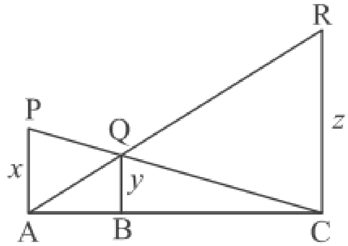


5. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio. (2024)

Answer. Correct figure, given, to prove and construction Correct proof

6. In the given figure PA, QB and RC are each perpendicular to AC. If (2024) AP = x, BQ = y and CR = z, then

prove that $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$



Answer.

$\Delta PAC \sim \Delta QBC$

$\therefore \frac{x}{y} = \frac{AC}{BC}$ or $\frac{y}{x} = \frac{BC}{AC}$ ---- (i)

$\Delta RCA \sim \Delta QBA$

$\therefore \frac{z}{y} = \frac{AC}{AB}$ or $\frac{y}{z} = \frac{AB}{AC}$ ---- (ii)

Adding (i) and (ii)

$$\frac{y}{x} + \frac{y}{z} = \frac{BC+AB}{AC}$$

$$\Rightarrow \frac{1}{x} + \frac{1}{z} = \frac{1}{y}$$

6.2 Similar Figures

VSA (1 mark)

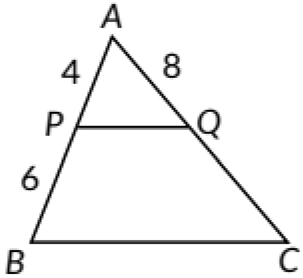
1. All concentric circles are ________ to each other. (2020)

2. Two polygons having same number of sides and corresponding sides proportional are similar or not? (Board Term 1, 2016)

6.3 Similarity of Triangles

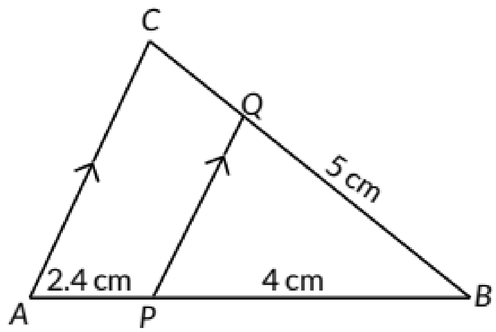
MCQ

3. In AABC, PQ||BC. If PB = 6 cm, AP = 4 cm, AQ = 8 cm, find the length of AC.
(a) 12 cm
(b) 20 cm
(c) 6 cm
(d) 14 cm



(2023)

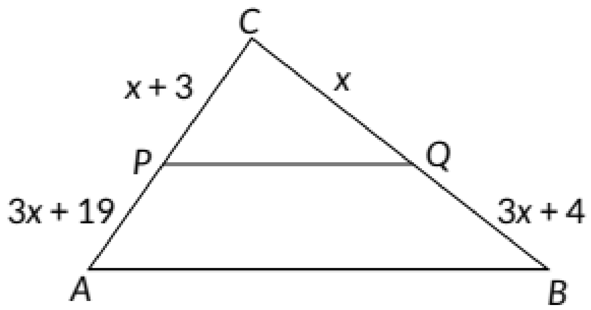
4. In the given figure, PQ II AC. If BP = 4 cm, AP = 2.4 cm and BQ = 5 cm, then length of BC is



(a) 8 cm (b) 3 cm

(c) 0.3 cm (d) $\frac{25}{3}$ cm (2023)

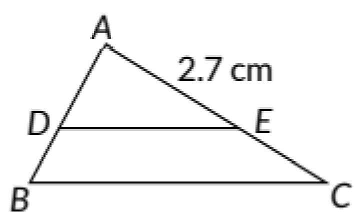
5. In the figure given below, what value of x will make PQ || AB?



(a) 2
(c) 4
(b) 3
(d) 5 (Term 1, 2021-22)

6.

In figure, $DE || BC$. If $\frac{AD}{DB} = \frac{3}{2}$ and $AE = 2.7$ cm, then EC is equal to

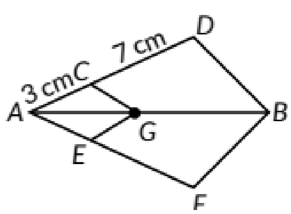


(a) 2.0 cm
(b) 1.8 cm
(c) 4.0 cm
(d) 2.7 cm

VSA (1 mark)

(2020) Ap

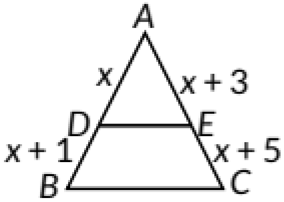
7. In figure, GC||BD and GE||BF. If AC = 3 cm and $CD = 7$ cm, then find the value of $\frac{AE}{AF}$. *(2019C)* Ap



8. In AABC, X is middle point of AC. If XY||AB, then prove that Y is middle point of BC. (Board Term I, 2017)

9. In AABC, D and E are point on side AB and AC respectively, such that DE || BC. If AE = 2 cm, AD = 3 cm and BD = 4.5 cm, then find CE. (Board Term I, 2017)

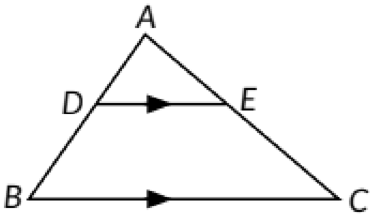
10. In AABC, DE || BC, then find the value of x.



(Board Term I, 2017)

11. In given figure, DE ||BC

If $\frac{AD}{DB}=\frac{3}{4}$ and AC = 14 cm, find EC.

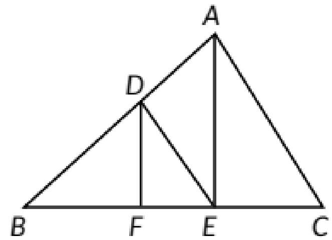


(Board Term I, 2017)

SAI (2 marks)

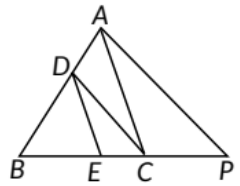
12. In the given figure, DE || AC and DF || AE.

Prove that $\frac{BF}{FE}=\frac{BE}{EC}$.



(NCERT, 2020)

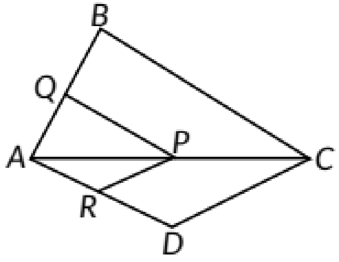
In figure, $DE \parallel AC$ and $DC \parallel AP$. Prove that $\frac{BE}{EC}=\frac{BC}{CP}$.



(2020) Ap

13. In figure, if PQ || BC and PR || CD, prove that

$\frac{QB}{AQ} = \frac{DR}{AR}$

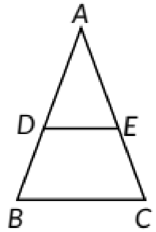


(2020)

SA II (3 marks)

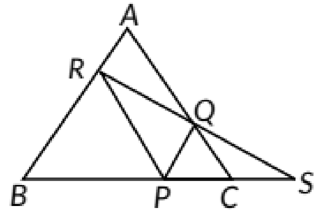
14.

In figure $\angle D = \angle E$ and $\frac{AD}{DB} = \frac{AE}{EC}$, prove that ΔABC is an isosceles triangle.



(2020) Ap

15. In the figure, P is any point on side BC of AABC. PQ || BA and PR || CA are drawn. RQ is extended to meet BC produced at S. Prove that SP2 = SB × SC.



(Board Term I, 2017)

LA (4/5/6 marks)

16. If a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, prove that the other two sides are divided in the same ratio. (2020, 2015)

OR

State and prove Basic Proportionality Theorem (Thales Theorem).(Board Term 1, 2015)

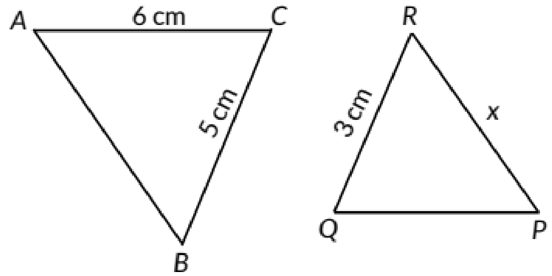
17. ABCD is a trapezium with AB||CD. E and F are points on non parallel sides AD and BC respectively, such

that $EF||AB$. Show that $\frac{AE}{ED}=\frac{BF}{FC}$ (2019C)

6.4 Criteria for Similarity of Triangles

MCQ

18. In the given figure, AABC ~ AQPR. If AC = 6 cm, BC= 5 cm, QR = 3 cm and PR = x, then the value of x is

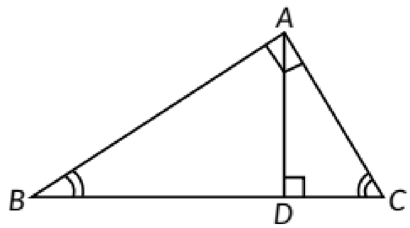


(a) 3.6 cm
(c) 10 cm
(b) 2.5 cm
(d) 3.2 cm (2023)

19. If AABC and APQR are similar triangles such that ZA = 31° and ZR = 69°, then ZQ is
(a) 70°
(b) 100°
(c) 90°
(d) 80° (Term I, 2021-22)

20. A vertical pole of length 19 m casts a shadow 57 m long on the ground and at the same time a tower casts a shadow 51 m long. The height of the tower is
(a) 171m
(b) 13 m
(c)17 m
(d) 117 m (Term I, 2021-22)

21. In the given figure, ZABC and ZACB are complementary to each other and ADI BC. Then,



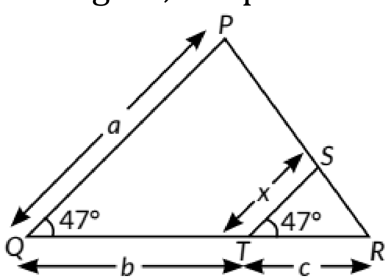
(a) BD.CD = BC2

(c) BD-CD = AD2

(b) AB-BC= BC2

(d) AB-AC = AD2 (Term I, 2021-22)

22. In the given figure, x expressed in terms of a, b, c, is



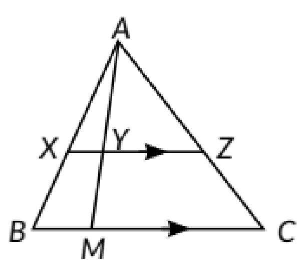
(a) $x = \frac{ab}{a+b}$ (b) $x = \frac{ac}{b+c}$

(c) $x = \frac{bc}{b+c}$ (d) $x = \frac{ac}{a+c}$

(Term I, 2021-22)

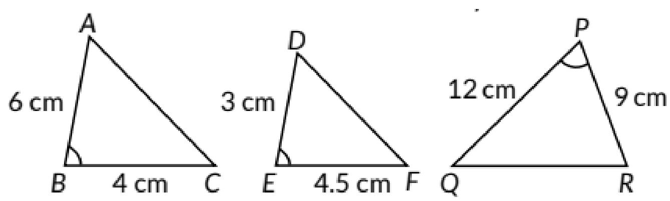
SAI (2 marks)

23. In the given figure, XZ is parallel to BC. AZ = 3 cm, ZC = 2 cm, BM = 3 cm and MC = 5 cm. Find the length of XY.23. In the given figure, XZ is parallel to BC. AZ = 3 cm, ZC = 2 cm, BM = 3 cm and MC = 5 cm. Find the length of XY.



(2023)

25. State which of the two triangles given in the figure are similar. Also statthe similarity criterion used.

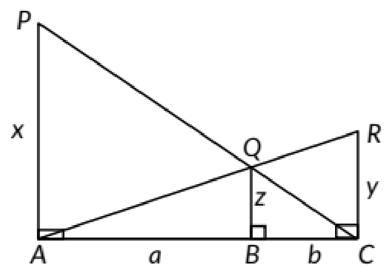


(Board Term I, 2016)

26. Sides AB, BC and median AD of a ΔABC are respectively proportional to sides PQ, QR and median PM of ΔPQR. Show that $\Delta ABC \sim \Delta PQR$. (Board Term 1, 2015)

SA II (3 marks)

27. PA, QB and RC are each perpendicular to AC. If $AP = x$, $QB = z$, $RC = y$, $AB = a$ and $BC = b$, then prove that $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$.

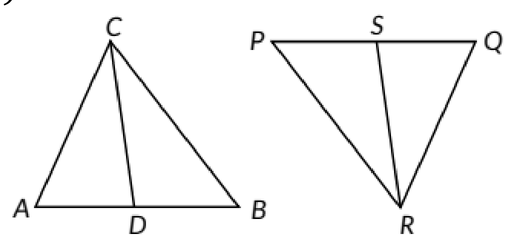


(2023)

28. In the given figure, CD and RS are respectively the medians of ΔABC and ΔPQR. If $\Delta ABC \sim \Delta PQR$ then prove that:

(i) $\Delta ADC \sim \Delta PSR$

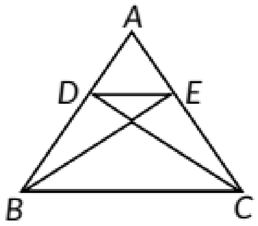
(ii) AD × PR= ACX PS



(2023)

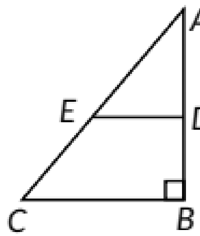
29. In the figure, if $\Delta BEA = \Delta CDA$, then prove that

$\Delta DEA - \Delta BCA$.



(Board Term I, 2017)

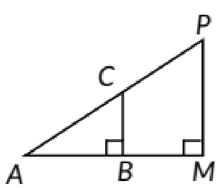
30. In ΔABC, $\angle ADE = \angle B$ then prove that $\Delta ADE \sim \Delta ABC$ also if AD = 7.6 cm, BD = 4.2 cm and BC = 8.4 cm, then
find DE.



(Board Term I, 2017)

31. A girl of height 100 cm is walking away from the base of a lamp post at a speed of 1.9 m/s. If the lamp is 5 m above the ground, find the length of her shadow after 4 seconds. (Board Term 1, 2016)

32. AABC and AAMP are two right angled triangles right angled at B and M respectively. Prove that CA × MP
= PAX BC.



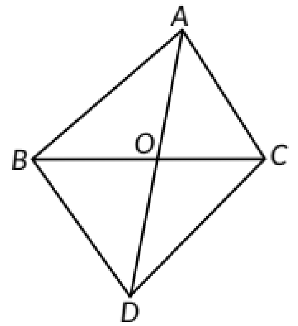
(Board Term I, 2016)

LA (4/5/6 marks)

33. (A) In a APQR, N is a point on PR, such that $QN \perp PR$. If
$PN \times NR = QN^2$, prove that /PQR = 90°. (2023)

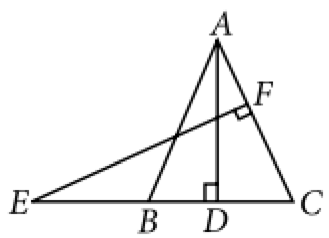
34. In the given figure, AABC and ADBC are on the same base BC. If AD intersects BC at O, prove that

$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$.



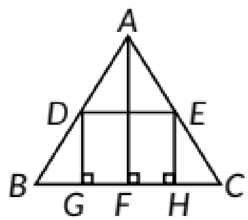
(2023)

35. In the given figure, E is a point on CB produced of an isosceles ΔABC, with side AB = AC. If AD ⊥ BC and EF ⊥ LAC, prove that ΔABD ~ ΔAECF.



(NCERT, AI 2019) U

36. In the given figure, ABC is a triangle and GHED is a rectangle. BC = 12 cm, HE = 6 cm, FC = BF and altitude AF = 24 cm. Find the area of the rectangle.



(Board Term I, 2017)

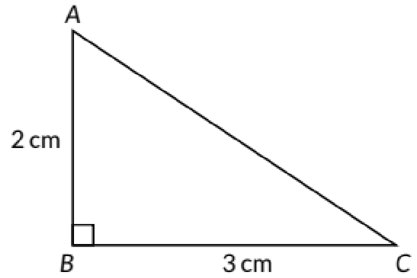
37. Two poles of height 'p' and 'q' metres are standing vertically on a level ground, 'a' metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is given by $\frac{pq}{p+q}$. *(Board Term I, 2017)*

38. In ΔABC, from A and B altitudes AD and BE are drawn. Prove that ΔADC ~ ΔBEC. Is ΔADB ~ ΔAEB and ΔADB ~ ΔADC? (Board Term 1, 2016)

Pythagoras Theorem

MCQ

39. Assertion (A): The perimeter of AABC is a rational number.
Reason (R): The sum of the squares of two rational numbers is always rational.



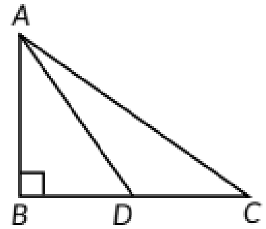
(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(c) Assertion (A) is true, but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true. (2023)

VSA (1 marks)

40. Aman goes 5 metres due west and then 12 metres due North. How far is he from the starting point? (2021 C)

SA II (3 marks)

41. In ΔABC, ZB = 90° and D is the mid point of BC. Prove that $AC^2 = AD^2+3CD^2$

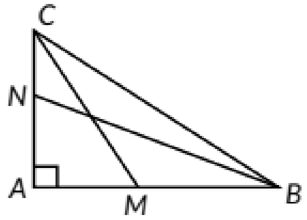


(2019)

42. Prove that the sum of squares of the sides of a rhombus is equal to the sum of squares of its diagonals. (2019)

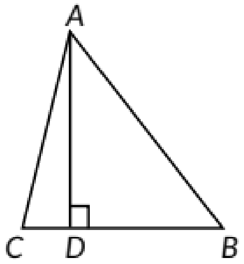
LA (4/5/6 marks)

43. In given figure BN and CM are medians of a right angled at A. Prove that 4 $(BN^2 + CM^2) = 5BC^2$



(2020C

44. The perpendicular from A on the side BC of a ΔABC intersects BC at D, such that DB = 3CD. Prove that
$2AB^2 = 2AC^2 + BC^2$.



(2019C)

CBSE Sample Questions

6.3 Similarity of Triangles

VSA (1 mark)

1. In the ΔABC, D and E are points on side AB and AC respectively such that DE ||BC. If AE = 2 cm, AD = 3 cm and BD = 4.5 cm, then find CE. (2020-21)

LA (4/5/6 marks)

2. Prove that if a line is drawn parallel to one side of a triangle intersecting the other two sides in distinct points, then the other two sides are divided in the same ratio.
Using the above theorem prove that a line through the point of intersection of the diagonals and parallel to the base of the trapezium divides the non parallel sides in the same ratio. (2022-23)

6.4 Criteria for Similarity of Triangles

MCQ

3. ΔABC-ΔPQR. If AM and PN are altitudes of ΔABC and ΔPQR respectively and $\Delta AB2: PQ^2 = 4 : 9$, then AM: PN=

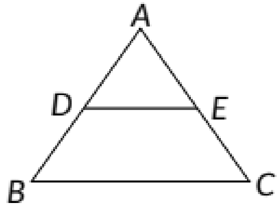
(a) 3:2
(b) 16:81
(c) 4:9
(d) 2:3 (2022-23)

OR

$\Delta ABC \sim \Delta PQR$. If AM and PN are altitudes of ΔABC and ΔPQR respectively and $AB^2: PQ^2 = 4 : 9$, then
AM: PN=

(a) 16:81
(b) 4:9
(c) 3:2
(d) 2:3 (Term I, 2021-22)

In the figure, if DE || BC, AD = 3 cm, BD = 4 cm and BC= 14 cm, then DE equals

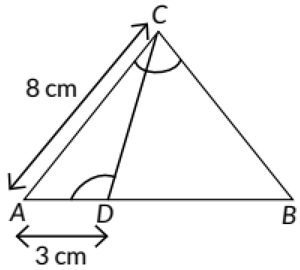


(a) 7 cm
(b) 6 cm
(c) 4 cm
(d) 3 cm (Term 1, 2021-22)

5. ΔABC is such that AB = 3 cm, BC = 2 cm, CA = 2.5 cm. If $\Delta ABC \sim \Delta DEF$ and EF = 4 cm, then perimeter of ΔDEF is

(a) 7.5 cm
(b) 15 cm
(c) 22.5 cm
(d) 30 cm (Term I, 2021-22)

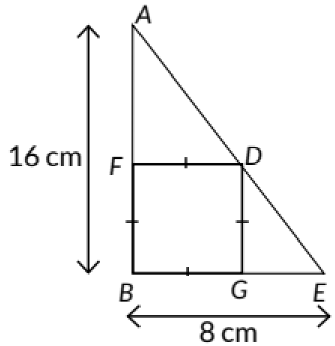
6. In the given figure, <ZACB = <CDA, AC = 8 cm,
AD = 3 cm, then BD is



(a) $\frac{22}{3}$ cm (b) $\frac{26}{3}$ cm (c) $\frac{55}{3}$ cm (d) $\frac{64}{3}$ cm

(Term I, 2021-22) An

7. Sides AB and BE of a right triangle, right angled at B are of lengths 16 cm and 8 cm respectively. The length of the side of largest square FDGB that can be inscribed in the triangle ABE is



(a) $\frac{32}{3}$ cm (b) $\frac{16}{3}$ cm (c) $\frac{8}{3}$ cm (d) $\frac{4}{3}$ cm

(Term I, 2021-22)

Case study-based questions are compulsory. Attempt any 4 sub parts. Each question carries 1 mark.

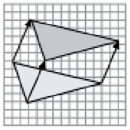
8. SCALE FACTOR AND SIMILARITY

Scale Factor

A scale drawing of an object is the same shape as the object but a different size. The scale of a drawing is a comparison of the length used on a drawing to the length it represents. The scale is written as a ratio.

Similar Figures

The ratio of two corresponding sides in similar figures is called the scale factor.



Translation or Slide

Hence, two shapes are Similar when one can become the other after a resize, flip, slides or turn.

(i) A model of a boat is made on the scale of 1: 4. The model is 120 cm long. The full size of the boat has a width of 60 cm. What is the width of the scale model?

(a) 20 cm (b) 25 cm (c) 15 cm (d) 240 cm

(ii) What will effect the similarity of any two polygons?

(a) They are flipped horizontally

(b) They are dilated by a scale factor

(c) They are translated down

(d) They are not the mirror image of one another

(iii) If two similar triangles have a scale factor of a: b. Which statement regarding the two triangles is true?

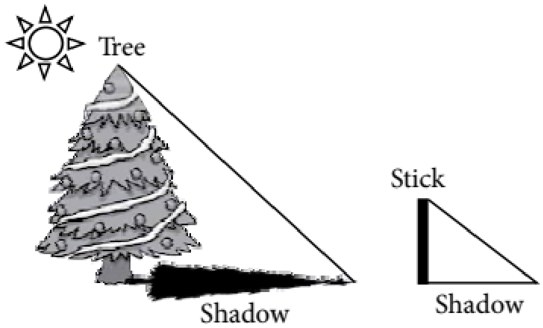
(a) The ratio of their perimeters is 3a: b

(b) Their altitudes have a ratio a: b

(c) Their medians have a ratio $\frac{a}{2} : b$

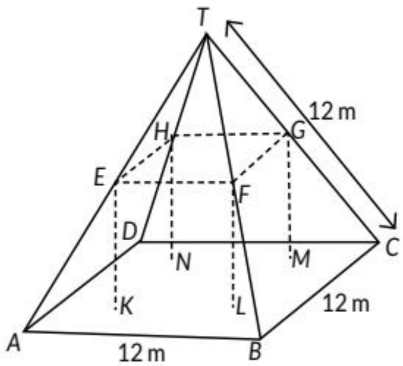
(d) Their angle bisectors have a ratio a^2: b2

(iv) The shadow of a stick 5 m long is 2 m. At the same time the shadow of a tree 12.5m high is



(a) 3m (b) 3.5 m (c) 4.5m (d) 5m

(v) Below you see a student's mathematical model of a farmhouse roof with measurements. The attic floor, ABCD in the model, is a square. The beams that support the roof are the edges of a rectangular prism, EFGHKLMN. E is the middle of AT, F is the middle of BT, G is the middle of CT, and H is the middle of DT. All the edges of the pyramid in the model have length of 12 m.



What is the length of EF, where EF is one of the horizontal edges of the block?

(a) 24 m

(c) 6m

(b) 3m

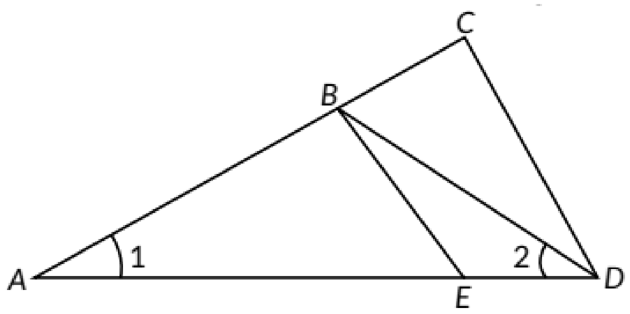
(d) 10 m (2020-21)

SAI (2 marks)

9.

In the given figure below, $\frac{AD}{AE}=\frac{AC}{BD}$ and $\angle 1 = \angle 2$.

Show that $\Delta BAE \sim \Delta CAD$.



(2022-23)

SA II (3 marks)

10. The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of the first triangle is 9 cm, find the length of the corresponding side of the second triangle. (2020-21)

SOLUTIONS

Previous Years' CBSE Board Questions

1. All concentric circles are similar to each other.

2. Two polygons having same number of sides and corresponding sides proportional are not similar.

3. (b): Since, PQ||BC

$\therefore \quad \frac{AP}{PB}=\frac{AQ}{QC}$ [By Thales theorem]

$\Rightarrow \quad \frac{4}{6}=\frac{8}{QC} \Rightarrow QC=\frac{8\times 6}{4}=12\text{ cm}$

4. (a): Since, PQ || AC.

$$\therefore \quad \frac{BQ}{QC} = \frac{BP}{AP}$$

$$\Rightarrow \quad \frac{5}{QC} = \frac{4}{2.4} \qquad [\because BP = 4 \text{ cm}, AP = 2.4 \text{ cm and } BQ = 5 \text{ cm}]$$

$$\Rightarrow \quad QC = \frac{5 \times 2.4}{4} = 3 \text{ cm}$$

$$\therefore \quad BC = BQ + QC = 5 + 3 = 8 \text{ cm}$$

5. (a): Suppose PQ || AB

:- By Basic Proportionality theorem, we have

$$\frac{CP}{PA} = \frac{CQ}{QB} \Rightarrow \frac{x+3}{3x+19} = \frac{x}{3x+4}$$

$$\Rightarrow \quad 3x^2 + 19x = 3x^2 + 9x + 4x + 12$$

$$\Rightarrow \quad 6x = 12 \Rightarrow x = 2$$

So, for $x = 2$, $PQ \parallel AB$.

6. (b): In AABC, DE || BC

$$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC} \qquad \text{[By B.P.T.]}$$

$$\Rightarrow \quad \frac{3}{2} = \frac{2.7}{EC} \qquad \text{[Given]}$$

$$\Rightarrow \quad 3EC = 2 \times 2.7$$

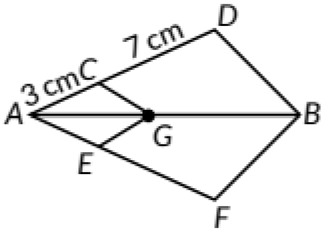
$$\Rightarrow \quad EC = \frac{5.4}{3} = 1.8 \text{ cm}$$

7. Here in the given figure,

GC || BD and GE || BF

AC = 3 cm and CD = 7 cm

By Basic Proportionality theorem,



we get $\frac{AC}{CD} = \frac{AE}{EF}$

$$\therefore \quad \frac{AE}{EF} = \frac{3}{7} \Rightarrow \frac{AF}{AE} = \frac{7}{3} \Rightarrow \frac{AE+EF}{AE} = \frac{3+7}{3}$$

$$\Rightarrow \quad \frac{AF}{AE} = \frac{10}{3}$$

$$\therefore \quad \frac{AE}{AF} = \frac{3}{10}$$

8. Given, X is middle point of AC and XY || AB. We have to prove Y is middle point of BC. In AABC, XY || AB

$\therefore \quad \frac{CX}{XA} = \frac{CY}{YB}$ [By B.P.T.] ...(i)

But X is the middle point of AC

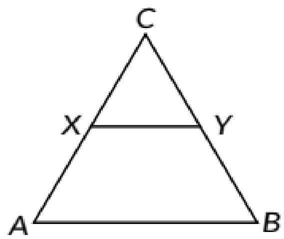
$\Rightarrow \quad CX = XA$ ...(ii)

From (i) and (ii), we get

$1 = \frac{CY}{YB}$

$\Rightarrow \quad CY = YB$

$\therefore \quad Y$ is middle point of BC



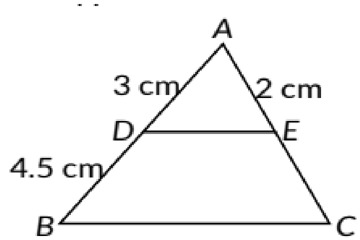
9.

In ΔABC, $DE \parallel BC$

$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$ [By B.P.T.]

$\Rightarrow \quad \frac{3}{4.5} = \frac{2}{EC}$

$\Rightarrow \quad 3EC = 9 \Rightarrow EC = 3$ cm



10.

In ΔABC, $DE \parallel BC$

$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$ [By B.P.T.]

$\Rightarrow \quad \frac{x}{x+1} = \frac{x+3}{x+5} \Rightarrow x(x+5) = (x+3)(x+1)$

$\Rightarrow \quad x^2 + 5x = x^2 + 3x + x + 3 \Rightarrow x = 3$

11.

In ΔABC, $DE \parallel BC$

$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$ [By B.P.T.]

$\Rightarrow \quad \frac{3}{4} = \frac{AC - EC}{EC}$ [Given]

$\Rightarrow \quad \frac{3}{4} = \frac{14 - EC}{EC}$

$\Rightarrow \quad 3EC = 56 - 4EC \Rightarrow 7EC = 56 \Rightarrow EC = 8$ cm

12.

In ΔABC, we have $DE \parallel AC$

$\Rightarrow \frac{BE}{EC} = \frac{BD}{DA}$ [By B.P.T] ...(i)

In ΔABE, $DF \parallel AE$

$\Rightarrow \frac{BF}{FE} = \frac{BD}{DA}$ [By B.P.T] ...(ii)

From (i) and (ii), we have $\frac{BF}{FE} = \frac{BE}{EC}$

Hence proved.

13.

In ΔABC, $PQ \parallel BC$

$\Rightarrow \frac{AQ}{QB} = \frac{AP}{PC}$ [By B.P.T.] ...(i)

In ΔACD, $PR \parallel CD$

$\Rightarrow \frac{AR}{DR} = \frac{AP}{PC}$ [By B.P.T.] ...(ii)

From (i) and (ii), $\frac{AQ}{QB} = \frac{AR}{DR} \Rightarrow \frac{QB}{AQ} = \frac{DR}{AR}$

Hence proved.

14.

Given, $\frac{AD}{DB} = \frac{AE}{EC}$ and $\angle D = \angle E$ ...(i)

We have to prove that ΔABC is an isosceles triangle.

Now, in ΔABC, $\frac{AD}{DB} = \frac{AE}{EC}$ [Given]

:- DE || BC

[By Converse of Basic Proportionality Theorem]

Also, <D= <B and ZE = /C [Corresponding angles] ...(ii)

From (i) and (ii), we get <B = <C ⇒ AB = AC

[Sides opposite to equal angles are equal]

:- AABC is an isosceles triangle.

15.

In ΔSRB, $PQ \parallel RB$

$\Rightarrow \frac{SP}{SB} = \frac{SQ}{SR}$ [By B.P.T.] ...(i)

Also, in ΔSPR, $PR \parallel QC$

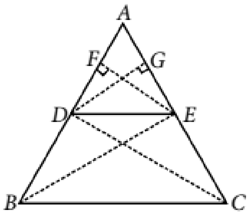
$\Rightarrow \frac{SC}{SP} = \frac{SQ}{SR}$ [By B.P.T.] ...(ii)

From (i) and (ii), we get,

$\frac{SP}{SB} = \frac{SC}{SP} \Rightarrow SP^2 = SB \times SC$

Hence proved.

16. Consider AABC in which DE || BC, DE intersects AB at D and AC at E.



To prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Join BE, CD and draw $EF \perp AB$, $DG \perp AC$.

Proof : Area of ΔEAD

$= \frac{1}{2} \times (\text{base} \times \text{height}) = \frac{1}{2} \times AD \times EF$

So, area(ΔEAD) = $\frac{1}{2} AD \times EF$

Again, area of $\Delta EDB = \frac{1}{2} \times (\text{base} \times \text{height}) = \frac{1}{2} \times DB \times EF$

So, area(ΔEDB) = $\frac{1}{2} DB \times EF$

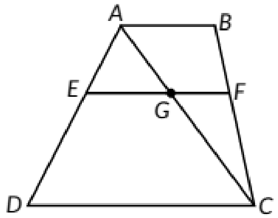
$\therefore \quad \frac{\text{area}(\Delta EAD)}{\text{area}(\Delta EDB)} = \frac{AD}{DB}$ ...(i)

Similarly, $\frac{\text{area}(\Delta EAD)}{\text{area}(\Delta ECD)} = \frac{AE}{EC}$ ...(ii)

Since, triangles EDB and ECD are on the same base DE and between the same parallel lines DE and BC.
So, area (ΔEDB) = area (ΔECD) ...(iii)

From (i), (ii) and (iii), we have $\frac{AD}{DB} = \frac{AE}{EC}$

17.



First join AC to intersect EF at G.
Given AB || DC and EF || AB
⇒ EF || DC ...(i)
[:: Lines parallel to same line are parallel to each other.]
Now in ΔADC, we have
EG||DC (:- EF||DC)

$$\Rightarrow \quad \frac{AE}{ED} = \frac{AG}{GC} \text{ (By } B.P.T.\text{)} \quad ...(ii)$$

Similarly in ΔCAB, we have

$$\frac{CG}{AG} = \frac{CF}{BF} \text{ (By } B.P.T.\text{)}$$

$$\Rightarrow \quad \frac{AG}{GC} = \frac{BF}{FC} \quad ...(iii)$$

From (ii) and (iii), we get

$$\frac{AE}{ED} = \frac{BF}{FC}$$

Hence proved.

18.

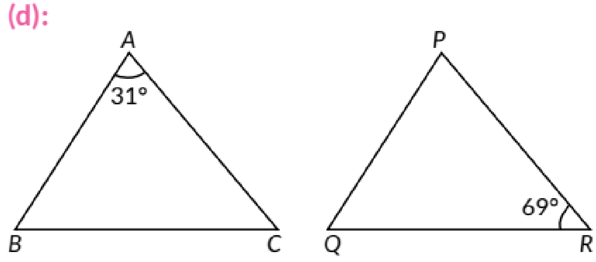
(b): Given, $\Delta ABC \sim \Delta QPR$

$$\therefore \quad \frac{AB}{QP} = \frac{BC}{PR} = \frac{AC}{QR} \quad \therefore \quad \frac{5}{x} = \frac{6}{3} \Rightarrow x = \frac{5 \times 3}{6} = 2.5$$

$\Rightarrow \quad x = 2.5$ cm

19.

(d):



Given, <ZA = 31°, <R = 69° and ΔABC ~ ΔPQR
:- <ZP = <A = 31°
:- <ZQ=180° - (31° + 69°) [By angle sum property]
= <ZQ=80°

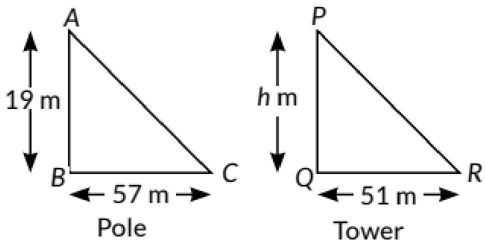
20. (c): Let AB be the pole and PQ be the tower.
Let height of tower be h m.
Now, ΔABC ~ ΔPQR

$\therefore \quad \frac{AB}{PQ}=\frac{BC}{QR}$

$\Rightarrow \quad \frac{19}{h}=\frac{57}{51}$

$\Rightarrow \quad h=\frac{19\times51}{57}$

$\Rightarrow \quad h = 17\text{ m}$



21. (c): We have,
<ABC+<ACB = 90° [Given] . (i)
In ΔBDA, <ABD+<BAD = 90° ... (ii) [By angle sum property]
From (i) and (ii), we get, <ACB = <BAD
In ΔADC and ΔBDA
<ADC=<BDA = 90°
<ACD=<BAD [Proved above]
:- ΔADC~ ΔBDA [By AA similarity]

22. (b): In ΔRST and ARPQ,
<R = <R [Common]
<RTS = <RQP = 47° [Given]
ΔRST ~ ΔRPQ [By AA similarity]

$\therefore \quad \frac{ST}{PQ}=\frac{RT}{RQ} \Rightarrow \frac{x}{a}=\frac{c}{c+b} \Rightarrow x=\frac{ac}{b+c}$

23. (a) Given, AZ = 3 cm, ZC = 2 cm, BM = 3 cm and MC = 5 cm
In ΔABC, XZ || BC

$\therefore \quad \frac{AX}{AB}=\frac{AY}{AM}=\frac{AZ}{AC}$ (Thales theorem) ...(i)

Now, AC = AZ + ZC=3+2 = 5 cm
BC= BM+MC=3+5=8 cm and

In ΔAXY and ΔABM

$\angle AXY = \angle ABM$ (Corresponding angles are equal, as $XZ \parallel BC$)

$\angle XAY = \angle BAM$ (Common)

$\therefore \quad \Delta AXY \sim \Delta ABM$ (By AA similarity criterion)

$$\therefore \quad \frac{AX}{AB} = \frac{XY}{BM} = \frac{AY}{AM} \quad \text{...(ii)}$$

(Corresponding sides of similar triangles.)

From (i) and (ii), we get $\frac{XY}{BM} = \frac{AZ}{AC}$

$$\Rightarrow \quad \frac{XY}{3} = \frac{3}{5}$$

$$\Rightarrow \quad XY = \frac{3 \times 3}{5} = \frac{9}{5} = 1.8 \text{ cm}$$

24. Given, PQ||BC

PQ = 3 cm, BC = 9 cm and AC = 7.5 cm

Since, PQ || BC

:- <APQ = <ABC (Corresponding angles are equal)

Now, in ΔAPQ and ΔABC

<APQ = <ABC (Corresponding angles)

<A = <A (Common)

ΔAPO - ΔABE (AA similarity)

$$\therefore \quad \frac{AP}{AB} = \frac{AQ}{AC} = \frac{PQ}{BC}$$

$$\therefore \quad \frac{AQ}{AC} = \frac{3}{9} \Rightarrow \frac{AQ}{7.5} = \frac{1}{3} \quad \Rightarrow \quad AQ = \frac{7.5}{3} = 2.5 \text{ cm}$$

25. In AABC and ADEF

$$\frac{AB}{EF} = \frac{6}{4.5} = \frac{60}{45} = \frac{4}{3}, \frac{BC}{DE} = \frac{4}{3}$$

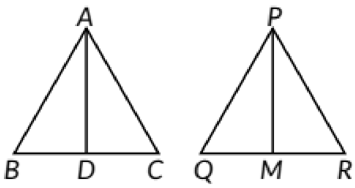
$\Rightarrow \quad \angle B = \angle E$

$\therefore \quad \Delta ABC \sim \Delta FED$ [By SAS similarity criterion]

26. We have ΔABC and ΔPQR in which AD and PM are medians corresponding to sides BC and QR respectively

such that $\frac{AB}{PQ}=\frac{BC}{QR}=\frac{AD}{PM}$

$\Rightarrow \quad \frac{AB}{PQ}=\frac{\frac{1}{2}BC}{\frac{1}{2}QR}=\frac{AD}{PM}$



$\Rightarrow \quad \frac{AB}{PQ}=\frac{BD}{QM}=\frac{AD}{PM}$

$\therefore$ Using SSS similarity, we have
$\Delta ABD \sim \Delta PQM$

$\therefore$ Their corresponding angles are equal.

$\Rightarrow \quad \angle ABD = \angle PQM \Rightarrow \angle ABC = \angle PQR$

Now, in ΔABC and ΔPQR, $\frac{AB}{PQ}=\frac{BC}{QR}$ ...(i)

Also, $\angle ABC = \angle PQR$ ...(ii)

From (i) and (ii),

$\Delta ABC \sim \Delta PQR$. [By SAS similarity]

27. Given: PA, QB and RC are each perpendicular to AC.

To prove : $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$

In ΔPAC and ΔQBC,

$\because \quad BQ || AP$

$\angle CPA = \angle CQB$ (Corresponding angles)

and $\angle PCA = \angle QCB$ (Common angles)

$\therefore \quad \Delta PAC \sim \Delta QBC$ (By AA similarity Criterion)

$\Rightarrow \quad \frac{AP}{BQ}=\frac{CA}{CB}=\frac{PC}{QC} \Rightarrow \frac{BQ}{AP}=\frac{CB}{CA}$

$\Rightarrow \quad \frac{z}{x}=\frac{CB}{CA}$ ...(i)

Now in ΔACR and ΔABQ, :: BQ||CR

<ARC=<AQB (Corresponding angles)

and <RAC = <QAB (Common)

:- ΔACR-ΔABQ

$\therefore \quad \frac{AC}{AB}=\frac{CR}{BQ}=\frac{AR}{AQ}$

$\therefore \quad \frac{BQ}{CR}=\frac{AB}{AC}$

$\Rightarrow \quad \frac{z}{y}=\frac{AB}{AC}$ ...(ii)

Adding (i) and (ii), we get

$$\frac{z}{x}+\frac{z}{y}=\frac{CB}{CA}+\frac{AB}{AC}$$

$$\Rightarrow \quad \frac{z}{x}+\frac{z}{y}=\frac{CB+AB}{AC} \Rightarrow \frac{z}{x}+\frac{z}{y}=\frac{AC}{AC}=1 \Rightarrow \frac{1}{x}+\frac{1}{y}=\frac{1}{z}$$

Hence proved.

28. (i) Given: Two similar triangles are ABC and PQR. CD and RS are medians of ΔABC and ΔPQR.

To prove : $\Delta ADC \sim \Delta PSR$

Proof : $\frac{CA}{RP}=\frac{AB}{PQ}$ $[\because \Delta ABC \sim \Delta PQR]$

$\Rightarrow \quad \frac{CA}{RP}=\frac{2AD}{2PS}$ (Since D and $R\,S$ are medians)

Now, In ΔADC and ΔPSR

$$\frac{CA}{RP}=\frac{AD}{PS}$$

and $\angle A = \angle P$ $(\because \Delta ABC \sim \Delta PQR)$

(By SAS similarity criterion)

$$\Delta ADC \sim \Delta PSR$$

Hence proved.

(ii) We have proved in part (i) $\Delta ADC \sim \Delta PSR$

$\therefore \quad \frac{AD}{PS}=\frac{DC}{SR}=\frac{AC}{PR}$

$\Rightarrow \quad \frac{AC}{AD}=\frac{PR}{PS} \Rightarrow AD \times PR = AC \times PS$

Hence proved.

29. We have, ΔBEA = ΔCDA

:- AB = AC and AE = AD [By C.P.C.T.]

$\Rightarrow \quad \frac{AB}{AD} = \frac{AC}{AE}$ ...(i)

Thus, in ΔDEA and ΔBCA, we have

$\frac{AB}{AD} = \frac{AC}{AE}$ [From (i)]

$\angle BAC = \angle DAE$ [Common]

$\therefore \quad \Delta DEA \sim \Delta BCA$ [By SAS similarity criterion]

30. In AADE and AABC

<ADE = <ABC [Given]

and <DAE = <BAC [Common]

:- ΔADE ~ ΔABC [By AA similarity criterion]

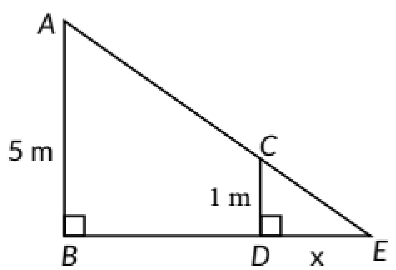
$\therefore \quad \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$

Now, $\frac{AD}{AB} = \frac{DE}{BC}$

$\Rightarrow \quad \frac{AD}{AD+BD} = \frac{DE}{BC} \quad \Rightarrow \quad \frac{7.6}{7.6+4.2} = \frac{DE}{8.4}$

$\Rightarrow \quad DE = \frac{7.6 \times 8.4}{11.8} = \frac{63.84}{11.8} = 5.4 \text{ cm}$

31. Let AB be the lamp post and CD be the girl's height and DE = x be the length of the shadow of the girl.



Here, CD = 100 cm = 1m, AB = 5m

Speed of girl = 1.9 m/s

We know that, Distance = Speed × Time

Distance of the girl from the lamp post after 4 seconds, BD = 1.9x4 = 7.6m

In ΔABE and ΔCDE

$\angle B = \angle D$ [Each 90°]
$\angle AEB = \angle CED$ [Common]
$\therefore \quad \Delta ABE \sim \Delta CDE$ [By AA similarity criterion]

$$\therefore \quad \frac{BE}{DE} = \frac{AB}{CD}$$

$$\Rightarrow \quad \frac{BD+DE}{DE} = \frac{AB}{CD} \qquad [\because BE = BD + DE]$$

$$\Rightarrow \quad \frac{7.6+x}{x} = \frac{5}{1}$$

We know that, Distance = Speed × Time
Distance of the girl from the lamp post after 4 seconds, BD = 1.9x4 = 7.6m
In ΔABE and ΔCDE

$\angle B = \angle D$ [Each 90°]
$\angle AEB = \angle CED$ [Common]
$\therefore \quad \Delta ABE \sim \Delta CDE$ [By AA similarity criterion]

$$\therefore \quad \frac{BE}{DE} = \frac{AB}{CD}$$

$$\Rightarrow \quad \frac{BD+DE}{DE} = \frac{AB}{CD} \qquad [\because BE = BD + DE]$$

$$\Rightarrow \quad \frac{7.6+x}{x} = \frac{5}{1}$$

$$\Rightarrow \quad 7.6 + x = 5x \Rightarrow 4x = 7.6 \Rightarrow x = 1.9$$

:- Length of shadow of girl after 4 seconds is 1.9 metres.

32. In ΔABC and ΔAMP
<ABC = <AMP [Each 90°]
<BAC = <PAM [Common]
:- ΔABE - ΔAMP [By AA similarity criterion]

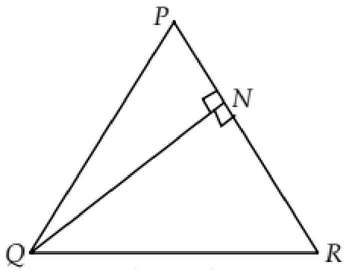
$$\therefore \quad \frac{CA}{PA} = \frac{BC}{MP} \qquad \text{[Corresponding sides of similar } \Delta\text{'s are proportional]}$$

$$\Rightarrow \quad CA \times MP = PA \times BC$$

33. Given, in ΔPQR, N is a point on PR, such that QN PR and PN. NR = QN^2
To prove: <PQR = 90°
Proof: We have, PN. NR = QN^2
⇒ PN . NR = QN . QN

$\Rightarrow \quad \frac{PN}{QN}=\frac{QN}{NR}$

In ΔQNP and ΔRNQ, $\frac{PN}{QN}=\frac{QN}{NR}$



and <QNP = <RNQ [each equal to 90°]

:- ΔANP~ΔRNQ [by SAS similarity criterion]

Then, ΔQNP and ΔRNQ are equiangulars.

i.e., <PQN= <QRN ...(i)

and <RQN = <QPN ...(ii)

Adding (i) and (ii), we get

<PQN + <RQN = <QRN+ <QPN

= <PQR = <QRN + <QPN ...(iii)

We know that, sum of angles of a triangle is 180°

In ΔPQR, <PQR + <QPR + <QRP = 180°

= <PQR + <QPN+ <QRN = 180°

[:: <QPR = <QPN and<ZQRP = <QRN]

<PQR+ZPQR = 180° [using (iii)]

2<PQR = 180°

$\Rightarrow \quad \angle PQR=\frac{180^\circ}{2}=90^\circ \quad \therefore \angle PQR = 90^\circ$

Hence proved.

34. We have, ΔABC and ΔDBC are on the same base BC.

Also, BC and AD intersects at O.

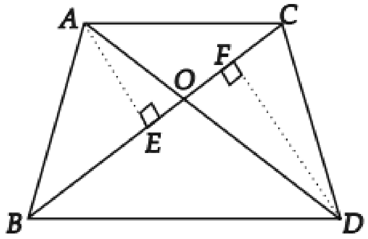
Let us draw AE ⊥ BC and DF ⊥ BC.

In ΔAOE and ΔDOF,

<AEO = <DFO = 90° ...(i)

Also, $\angle AOE = \angle DOF$ ...(ii)

[Vertically Opposite Angles]



:- From (i) and (ii), we get

$\Delta AOE \sim \Delta DOF$ [By AA similarity]

:- Their corresponding sides are proportional.

$$\Rightarrow \quad \frac{AE}{DF} = \frac{AO}{DO} \qquad ...(iii)$$

Now, $ar(\Delta ABC) = \frac{1}{2} BC \times AE$

And $ar(\Delta DBC) = \frac{1}{2} BC \times DF$

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2} BC \times AE}{\frac{1}{2} BC \times DF} = \frac{AE}{DF} \qquad ...(iv)$$

From (iii) and (iv), we have

$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}.$$

Hence Proved.

35. Given, ΔABC is an isosceles triangle with $AB = AC$

:- $\angle ABC = \angle ACB = \angle ABD = \angle ECF$ ...(i)

Now, in ΔABD and ΔECF

$\angle ADB = \angle EFC$ [Each 90°]

$\angle ABD = \angle ECF$ [Using (i)]

:- By AA similarity criterion, $\Delta ABD \sim \Delta ECF$.

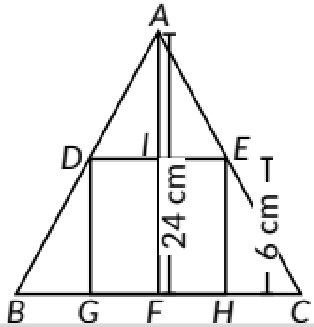
36. In ΔABC, $BC = 12$ cm,

$EH = DG = 6$ cm,

$BC = 12$ cm

$$\Rightarrow BF = FC = \frac{12}{2} = 6\,\text{cm}$$

and $AF = 24$ cm, $DE = GH$



Now, in ΔAFC and <EHC
<AFC = <EHC [Each 90°]
<ACF = <ECH [Common]
:- By AA similarity criterion,
ΔAFC ~ ΔEHC

$$\therefore \quad \frac{AF}{EH}=\frac{FC}{HC} \Rightarrow \frac{24}{6}=\frac{6}{HC} \Rightarrow HC=\frac{6\times 6}{24}=1.5\text{ cm}$$

Now, FHFC-HC = (6-1.5) cm = 4.5 cm
GH = 2 × FH = 2x4.5 = 9 cm
Area of rectangle DEHG=HEX GH = 6 x 9 = 54 cm^2

37. Let AB and CD be two poles
of height p and q metres
respectively and poles are 'a'
metres apart i.e., AC = a metres.
Let AD and BC meet at 'O' such
that OL = h metres
Let CL=x and LA = y
:- x+y=a
In ΔABC and ΔLOC, we have
<CAB = <CLO [Each 90°]
<C=<C [Common]
:- AABC - ALOC [By AA similarity criterion]

$$\therefore \quad \frac{CA}{CL}=\frac{AB}{OL}$$

$$\Rightarrow \frac{a}{x}=\frac{p}{h} \Rightarrow x=\frac{ha}{p} \qquad \text{...(i)}$$

In ΔALO and ΔACD, we have
∠ALO = ∠ACD [Each 90°]
∠A = ∠A [Common]
∴ ΔALO ~ ΔACD [By AA similarity criterion]

$$\therefore \frac{AL}{AC}=\frac{OL}{DC} \Rightarrow \frac{y}{a}=\frac{h}{q} \Rightarrow y=\frac{ah}{q} \qquad \text{...(ii)}$$

$$\text{Now, } x+y = ah\left(\frac{1}{p}+\frac{1}{q}\right) \qquad \text{[From (i) and (ii)]}$$

$$\Rightarrow x+y=ah\left(\frac{p+q}{pq}\right) \Rightarrow a=ah\left(\frac{p+q}{pq}\right) \Rightarrow h=\frac{pq}{p+q} \text{ metres.}$$

Hence, the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole

is $\frac{pq}{p+q}$ metres.

38. In ΔADC and ΔBEC

<DC = <BEC [Each 90°]

<ACD = <BCE [Common]

:- ΔADC ~ ΔBEC [By AA similarity criterion]

In ΔADB and ΔAEB

<ADB = <AEB [Each 90°]

<BAD+<EAB

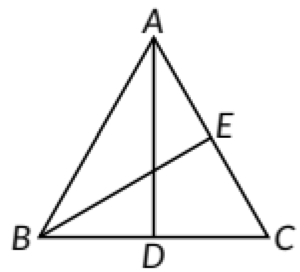
:- ΔADB is not similar to ΔAEB.

Again, in ΔADB and ΔADC

<BAD + <DAC

<ADB = <ADC [Each 90°]

:- ΔADB is not similar to ΔADC



39. (d): In AABC, $AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = 2^2+32$

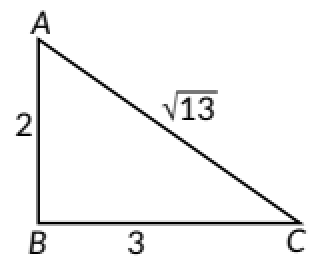
$\Rightarrow AC^2=4+9$

$\Rightarrow AC=\sqrt{13}$ cm

So, perimeter is $(2+3+\sqrt{13})$ cm

$=(5+\sqrt{13})$ cm which is irrational.

Hence, Assertion in false but Reason is true.



40. Let Aman starts from A point and continues 5 m towards west and reached at B point, from which he goes 12 m towards North reached at C point finally.

In AABC, we have

$AC^2 = AB^2 + BC^2$

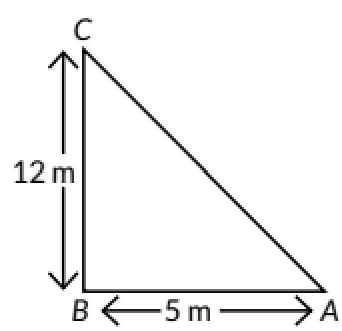
$AC^2 = 5^2 + 12^2$

(By Pythagoras theorem)

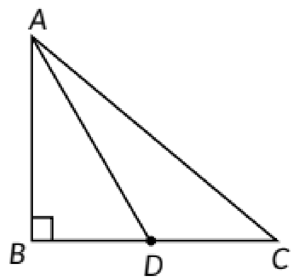
$AC^2 = 25+ 144 = 169$

$\Rightarrow AC = 13$ m

So, Aman is 13 m away from his starting point.



41.



In ΔABC, ZB = 90°

$AB^2 + BC^2 = AC^2$ (By Pythagoras theoram)

$\Rightarrow$ AB2 = $AC^2 - BC^2$ ...(i)

Similarly by pythagoras theoram, In ΔABD,

$AD^2 = AB^2 + BD^2$ ...(ii)

$\Rightarrow AB^2 = AD^2 - BD^2$ ...(iii)

From eqn (i) and (ii), we get $AC^2 - BC^2 = AD^2 - BD^2$

$\Rightarrow AC^2 = AD^2 - BD^2 + BC^2$

As D is the mid point of BC,

$\therefore \quad BD = CD$ or $CD = \frac{1}{2}BC \qquad [\because D$ is midpoint of $BC]$

AC^2 = AD2 - $CD^2 + BC^2$

AC^2 = AD2 - $CD^2 + (2CD)^2$ [::: BC= 2CD]

$\Rightarrow AC^2 = AD^2 + 3CD^2$

Hence proved.

42. Let we have a rhombus ABCD.

:- Diagonal of a rhombus bisect each other at right angles.

:- OA = OC and OB = OD

Also, <AOB = <BOC [Each = 90°]

and <COD = <DOA [Each = 90°]

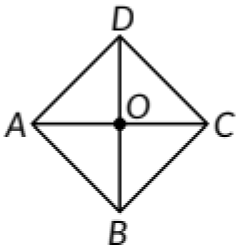
In ΔAOB, we have

$AB^2 = OA^2 + OB^2$ ...(i)

(By Pythagoras theorem)

Similarly in ABOC, we have

$BC^2 = OB^2 + OC^2$ ...(ii)



In ACOD,

$CD^2 = OC^2 + OD^2$ ...(iii)

In right ΔAOD

$DA^2 = OD^2 + OA^2$ ...(iv)

On adding (i), (ii), (iii) and (iv), we get

$AB^2 + BC^2 + CD^2 + DA^2$

$= [OA^2 + OB^2] + [OB^2 + OC^2] + [OC^2 + OD^2] + [OD^2 + OA^2]$

$= 2\,[OA^2 + OB^2 + OC^2 + OD^2]$

:- $OA^2 = OC^2$ and $OB^2 = OD^2$

$$\Rightarrow \quad AB^2 + BC^2 + CD^2 + DA^2 = 2[2OA^2 + 2OB^2]$$

$$= 2\left[2\left(\frac{1}{2}AC\right)^2 + 2\left(\frac{1}{2}BD\right)^2\right] \quad [\because O \text{ is mid point of } AC \text{ and } BD]$$

$$= 2\left[\frac{AC^2}{2} + \frac{BD^2}{2}\right] = AC^2 + BD^2$$

Thus, sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

43. In ΔABC, BN and CM are medians and $<A = 90°$

To prove : $4(BN^2 + CM^2) = 5BC^2$

In ΔABC, $ZA = 90°$

:- $BC^2 = AB^2 + AC^2$ ...(i) (By Pythagoras theorem)

In ΔCAM, $ZA = 90°$

:- $CM^2 = AC^2 + AM^2$

$$\Rightarrow \quad CM^2 = \left(\frac{1}{2}AB\right)^2 + AC^2 \qquad [\because M \text{ is midpoint of } AB]$$

$$\Rightarrow \quad CM^2 = \frac{1}{4}AB^2 + AC^2 \qquad ...(ii)$$

Now in ΔBAN, $\angle A = 90°$

$$\therefore \quad BN^2 = AN^2 + AB^2 \qquad \text{(By Pythagoras theorem)}$$

$$\Rightarrow \quad BN^2 = \left(\frac{1}{2}AC\right)^2 + AB^2 \qquad (\because N \text{ is midpoint of } AC)$$

$$\Rightarrow \quad BN^2 = \frac{1}{4}AC^2 + AB^2 \qquad ...(iii)$$

Add (ii) and (iii), we get

$$CM^2 + BN^2 = \frac{1}{4}AC^2 + \frac{1}{4}AB^2 + AB^2 + AC^2$$

$$\Rightarrow \quad 4(CM^2 + BN^2) = 5\,(AC^2 + AB^2) = 5BC^2 \qquad \text{(Using (i))}$$

Hence proved.

44. We have, ΔABC such that AD ⊥BC. ΔABC intersect BC at D such that BD = 3CD.
In right ΔADB, by Pythagoras theorem, we have
$AB^2 = AD^2 + BD^2$ ...(i)
Similarly in ΔACD, we have $AC^2 = AD2^2 + CD^2$ ...(ii)
Subtracting (ii) from (i), we get
AB2 - $AC^2 = BD^2 - CD^2$ ...(iii)
Now, BC= DB + CD = 4 CD [::: BD = 3CD]

$$\Rightarrow \quad CD = \frac{1}{4}BC$$

$$\therefore \quad BD = BC - CD = BC - \frac{1}{4}BC = \frac{3}{4}BC$$

Substituting the value of *BD* and *CD* in eqn.(iii) we get

$$AB^2 - AC^2 = \left[\frac{3}{4}BC\right]^2 - \left[\frac{1}{4}BC\right]^2$$

$$\Rightarrow \quad AB^2 - AC^2 = BC^2\left[\left(\frac{3}{4}\right)^2 - \left(\frac{1}{4}\right)^2\right]$$

$$= BC^2\left[\left(\frac{3}{4}+\frac{1}{4}\right)\left(\frac{3}{4}-\frac{1}{4}\right)\right] = BC^2\left[(1)\left(\frac{1}{2}\right)\right] = \frac{1}{2}BC^2$$

$$\Rightarrow \quad 2AB^2 - 2AC^2 = BC^2 \text{ or } 2AB^2 = 2AC^2 + BC^2$$

Hence proved.

CBSE Sample Questions

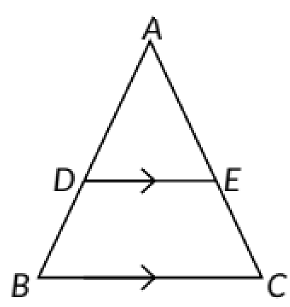
1.

In ΔABC, we have $DE \parallel BC$

$\therefore \quad \frac{AD}{BD} = \frac{AE}{CE}$ [By B.P.T.] **(1/2)**

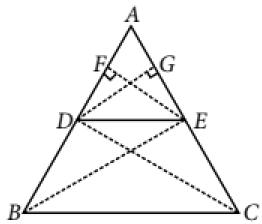
$\Rightarrow \quad \frac{3}{4.5} = \frac{2}{CE}$

$\Rightarrow \quad CE = 3$ cm **(1/2)**



2. Consider AABC in which DE || BC, DE intersects AB at D and AC at E.

To prove : $\frac{AD}{DB}=\frac{AE}{EC}$



Construction : Join BE, CD and draw $EF \perp AB$, $DG \perp AC$. **(1/2)**

Area of ΔEAD

$=\frac{1}{2}\times(\text{base}\times\text{height})=\frac{1}{2}\times AD\times EF$

So, area(ΔEAD) = $\frac{1}{2} AD \times EF$

Again, area of $\Delta EDB=\frac{1}{2}\times(\text{base}\times\text{height})$

$=\frac{1}{2}\times DB\times EF$ **(1)**

So, area(ΔEDB) = $\frac{1}{2} DB \times EF$

$\therefore \quad \frac{\text{area}(\Delta EAD)}{\text{area}(\Delta EDB)}=\frac{AD}{DB}$ ...(i)

Similarly, $\frac{\text{area}(\Delta EAD)}{\text{area}(\Delta ECD)}=\frac{AE}{EC}$ ...(ii) **(1)**

Since, triangles EDB and ECD are on the same base DE and between the same parallel lines DE and BC. So, area (ΔEDB) = area (ΔECD) ...(iii) (1/2)

From (i), (ii) and (iii), we have $\frac{AD}{DB}=\frac{AE}{EC}$ **(1/2)**

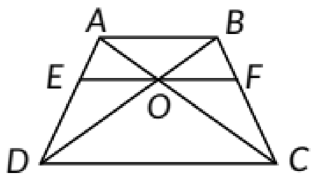
Using above theorem :

In ΔADB

Since $EO \parallel AB$

Using Basic Proportionality theorem

$\frac{AE}{DE}=\frac{BO}{DO}$ ...(i) **(1/2)**



In ΔBDC

Since $OF \parallel CD$

Using Basic Proportionality theorem

$\frac{BO}{DO}=\frac{BF}{FC}$ ...(ii) **(1/2)**

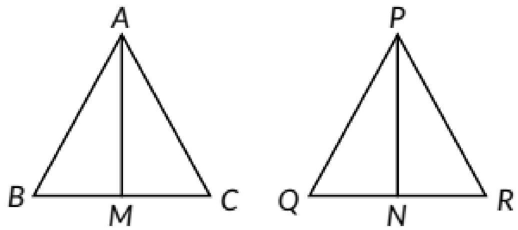
Comparing (i) and (ii), we get

$\frac{AE}{DE}=\frac{BF}{FC}$ **(1/2)**

Hence proved.

3.

(d):Given, $\frac{AB^2}{PQ^2}=\frac{4}{9} \Rightarrow \frac{AB}{PQ}=\frac{2}{3}$



$\because \quad \Delta ABC \sim \Delta PQR$

$\therefore \quad \frac{AB}{PQ}=\frac{BC}{QR}=\frac{AC}{PR}=\frac{AM}{PN}$

$\Rightarrow \quad \frac{AB}{PQ}=\frac{AM}{PN}=\frac{2}{3}$ **(1)**

4. (b): Since, DE || BC ⇒ AABC ~ AADE
(By AA similarity criterion)

So, $\frac{AD}{AB}=\frac{DE}{BC} \Rightarrow \frac{3}{7}=\frac{DE}{14} \Rightarrow DE=6$ cm **(1)**

5. (b): : AABC - ADEF

$\therefore \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF}=\frac{BC}{EF}$

$\Rightarrow \frac{3+2+2.5}{\text{Perimeter of } \Delta DEF}=\frac{2}{4}$

$\Rightarrow$ Perimeter of ΔDEF = 15 cm **(1**

6. (c): In ΔACD and AABC, we have
<A = <A (Common)
<CDA = ZBCA (Given)
:- ΔACD ~ ΔABC (By AA similarity)

$\therefore \frac{AC}{AB}=\frac{AD}{AC}$

$\Rightarrow \frac{8}{AB}=\frac{3}{8} \Rightarrow AB=\frac{64}{3}$ cm

So, $BD = AB - AD = \frac{64}{3}-3=\frac{55}{3}$ cm **(1)**

7. (b): AABE is a right triangle and FDGB is a square of side x cm (say).

In ΔAFD and ΔDGE,

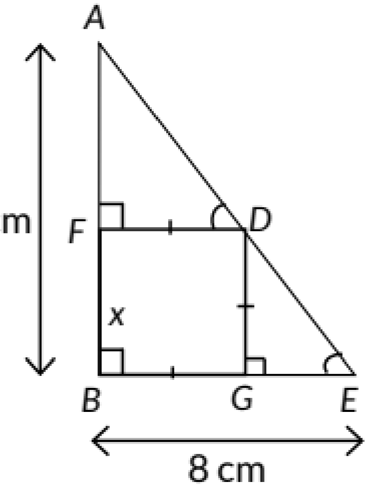
$\angle F = \angle G$ (Each 90°)

$\angle ADF = \angle DEG$

(Corresponding angles)

$\therefore \quad \Delta AFD \sim \Delta DGE$

(By AA similarity)



$$\therefore \frac{AF}{DG} = \frac{FD}{GE}$$

$$\Rightarrow \frac{16-x}{x} = \frac{x}{8-x}$$

$$\Rightarrow 128 - 24x + x^2 = x^2$$

$$\Rightarrow 128 = 24x \Rightarrow x = \frac{16}{3} \text{ cm}$$ **(1)**

8.

(i) (c) : Given scale factor = $\frac{1}{4}$ and width of full size of boat = 60 cm.

$$\therefore \frac{1}{4} = \frac{\text{Width of scale model}}{60}$$

$\Rightarrow$ Width of scale model = 15 cm. **(1)**

(ii) (d): They are not the mirror image of one another. (1)

(iii) (b): Their altitudes have a ratio a: b. (1)

(iv) (d): Since the two triangles are similar so the ratio of their corresponding sides are equal.

$$\therefore \frac{\text{Height of tree}}{\text{Shadow of tree}} = \frac{\text{Height of stick}}{\text{Shadow of stick}}$$

$$\Rightarrow \frac{12.5}{\text{Shadow of tree}} = \frac{5}{2}$$

$$\Rightarrow \text{Shadow of tree} = \frac{12.5 \times 2}{5} = 5 \text{ m}$$ **(1)**

(v) (c): Since E is the middle point of AT and F is the middle point of BT.

So, $ET = \frac{AT}{2} = \frac{12}{2} = 6$ m and $FT = \frac{BT}{2} = \frac{12}{2} = 6$ m

Now, $\Delta ETF \sim \Delta ATB$ [By SAS similarity criterion]

$\Rightarrow \quad \frac{ET}{AT} = \frac{FT}{BT} = \frac{EF}{AB} \Rightarrow EF = 6$ m **(1)**

9.

In ΔABD,

$\angle 1 = \angle 2$

$\therefore \quad BD = AB$ ...(i)

Given, $\frac{AD}{AE} = \frac{AC}{BD}$

Using equation (i), we get

$\frac{AD}{AE} = \frac{AC}{AB}$ ...(ii) **(1)**

In ΔBAE and ΔCAD, by equation (ii),

$\frac{AC}{AB} = \frac{AD}{AE}$

$\angle A = \angle A$ (common)

$\therefore \quad \Delta BAE \sim \Delta CAD$ [By SAS similarity criterion] **(1)**

10. Given one side of first triangle is 9 cm.
Let the length of the corresponding side of the second triangle be x cm. (1)
Now, ratio of perimeter

$$= \frac{\text{Perimeter of first triangle}}{\text{Perimeter of second triangle}} = \frac{9}{x}$$ **(1/2)**

[:- In similar triangles, the perimeter of the triangle will be in the ratio of their corresponding sides.]

$\Rightarrow \quad \frac{25}{15} = \frac{9}{x}$ **(1/2)**

$\Rightarrow \quad x = 5.4$ cm **(1)**